Why do we need HetNets?

Preliminaries
A Continuous LP
Converse
Utility Scheduling - Preliminaries
α-fair Utility Scheduling

Capacity and Scheduling in Heterogeneous Networks

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Talk Summary

- Mobile Radio and the Spectrum Crunch
- Getting more Capacity and How much do we Have?
- Utility Schedulers
- Closing Remarks
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Mobiles Past

An Entrepreneur Securing a Deal using an Early Mobile Phone
Mobiles Present

Progress toward Data, Apps - Location Based Information
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Mobiles Future

Future User having Trouble with a Hotel Booking ....
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Talk Summary

- A SnapShot Resource Allocation Problem
- A Continuous LP
- Capacity and Scheduling
- $\alpha$-fair Utility Scheduling
- Stability Results
- Conclusions
Gaining Capacity using HetNets

- Small cells (pico/femto) to increase frequency reuse
  - Place in areas of poor coverage
  - Areas of traffic concentration - ”Hot Spots”
- Adapt Network to Match Traffic Load
A Simplified HetNet Model

$L = 4$ picos - all users in range of macro and at most one pico
No Interference between Pico Cells
Flexible Allocation

- **Time Share Spectrum**
  - Macro Cell/Pico Cells
  - Use Almost Blanking SubFrames (fine granularity)
- **Cell Range Expansion for Picos**
  - Expand to cover more mobiles
  - Contract and send at Higher Rate

For following, see [1]

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ABS Frames and Time Sharing

1 time unit = macro time allocation + pico time allocation
Mobiles can TimeShare Macro/Pico (Split)
Empty the Network!!

\[ D_{1,n} = x_{3,n} + y_{3,n} \text{ bits} \]

\[ y_{3,n} \text{ bits} \]

\[ S_{3,n} \]

\[ x_{3,n} \text{ bits} \]

\[ R_{3,n} \]

\[ 0 \]
The problem to be solved is the following linear program:

\[
\begin{align*}
\text{min} & \quad f + \sum_{l=0}^{L} \sum_{n=1}^{N_l} \frac{y_{l,n}}{S_{l,n}} \\
\text{sub} & \quad \sum_{n=1}^{N_l} \frac{x_{l,n}}{R_{l,n}} \leq f \quad \forall l \\
& \quad x_{l,n} + y_{l,n} \geq D_{l,n} \quad \forall l, \forall n = 1, 2, \ldots N_l \\
& \quad f \geq 0, x_{l,n} \geq 0, y_{l,n} \geq 0 \quad \forall l, \forall n = 1, 2, \ldots N_l
\end{align*}
\]

where \( f \) is the time allocated to the picocells.
Solution Structure

- $\rho_{l,n} := \frac{R_{l,n}}{S_{l,n}}$
- Order User - Decreasing in $\rho$
- Large $\rho \rightarrow$ pico, Small $\rho \rightarrow$ macro, $= \rho \rightarrow$ Split

\[ m_j = 4 \quad \text{Pico cell } j \]
\[ N_j = 6 \]
Let’s make the Problem Continuous …
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Continuous LP parameters

\[ \lambda_S \eta (d \xi) = \lambda (d \xi), \eta (d \xi) \text{ probability density} \]

\[ R_\ell(\xi), S_\ell(\xi) \text{ Phy. Rates Pico/Macro - Pico } \ell \]

\[ x_\ell(\xi), y_\ell(\xi) \text{ bit assignments at location } \xi \]

\[ D \text{ download file size (could be random, here fixed)} \]

Largest $\lambda_S$ for which network is stable?
Continuous LP

\[
\begin{align*}
\min & \quad \tau = f + \sum_{\ell=1}^{L} \int \frac{y_{\ell}(\xi)}{S_{\ell}(\xi)} \lambda(d\xi) \\
\text{sub} & \quad \int \frac{x_{\ell}(\xi)}{R_{\ell}(\xi)} \lambda(d\xi) \leq f \quad \forall \ell
\end{align*}
\]

where,

\[y_{\ell}(\xi) = D - x_{\ell}(\xi)\]

is the file constraint
Optimal solution
For some $\rho_1, \cdots, \rho_L > 0$,
\[
x_\ell^*(\xi) = \begin{cases} 
D & \frac{R_\ell(\xi)}{S_\ell(\xi)} \geq \rho_\ell \\
0 & \frac{R_\ell(\xi)}{S_\ell(\xi)} < \rho_\ell
\end{cases} 
\]
\[(3)\]

\[
f^* = \max_\ell \int \frac{x_\ell^*(\xi)}{R_\ell(\xi)} d\xi
\]

If $\tau^* < 1$, $\exists$ a stable schedule $\cdots$
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Bundling $[nT, (n + 1)T)$, $n \in \mathbb{N}_0$
**Bundling Algorithm**

1. $B := 1$, Wait until $n_B = 1$
2. Serve bundle $B$, starting $n_B T$
3. Let $f_B T$ completion slot for bundle $B$
4. $B := B + 1, n_B := \max\{f_B, B\}$
5. Go to 2
Bundling defines a $D/G/1$ queue, bundle delay $= W_n$

$\tau < 1$, assumptions $\rightarrow \mathbb{E}[W_n]$ Uniformly Bounded

$W_n$ satisfies Spitzer's identity,

$$
\mathbb{E}[W_n] = \mathbb{E}\left[\max_{k \leq n} S_k^+\right] = \sum_{k=1}^{n} \frac{1}{k} \mathbb{E}\left[S_k^+\right]
$$

$S_k = X_k - kT$, $X_k$ duration first $k$ bundles
SLLN and Stability

Any bounded, measurable \( v : S \to \mathbb{R}_+ \),

\[
\frac{1}{T} \sum_{n=1}^{N_T} v(\xi_n(\omega)) \to \int_S v(\xi) \lambda(d\xi)
\]

(4)
a.s. and in \( L_1 \).

\( v^T_\ell(\omega) \equiv \frac{1}{T} \sum_{n=1}^{N_T} \frac{x_\ell(\xi_n(\omega))}{R_\ell(\xi_n(\omega))} \)

is UI, \( \ell = 1, \cdots , L \).

\[ \to f_T(\omega) = \max_\ell v^T_\ell(\omega) \]

is UI so that \( \mathbb{E}[f_T] \to f^* \)

\[
\mathbb{E}[f_T] + \sum_{\ell=0}^L \mathbb{E} \left[ \frac{1}{T} \sum_{n=1}^{N_T} \frac{y_\ell(\xi_n)}{S_\ell(\xi_n)} \right] \to \tau^* < 1
\]
A schedule $\pi$ is **clearing** if departure time $D_\pi^\omega(n) < \infty$, $a.s., \forall n$

**Prop (Hanly, W.)**

*Let $\tau^*$ be optimal solution to the LP. If $\tau^* < 1$, $\exists$ a clearing schedule $\pi$ with ergodic properties. Also define $S_n^\pi(\omega) :=$ sojourn time nth mobile, then $\pi$ satisfies,*

$$\mathbb{E}[S_n^\pi(\omega)] < \bar{S} < \infty$$  (5)
Converse Holds as Well!

Continuous LP $\tau^* > 1 \rightarrow$ No Stable Schedule
Let $\pi$ be any clearing schedule. Define $V^\pi_T(\omega)$ to be network time needed to clear mobiles arriving in $[0, T]$

**Prop (Hanly, W.)**

*Let $\tau^*$ be the solution to the continuous LP. Suppose that $\tau^* > 1$ then there is a fixed constant $\eta > 0$, such that for all $\pi$*

$$
\liminf_{T \to \infty} \frac{V^\pi_T(\omega)}{T} = 1 + \eta
$$

*almost surely.*
Proof Sketch I

Arrivals in \([0, T]\) supposed to arrive at time 0. Apply discrete LP with outcome \(V_T^{(LP)}(\omega)\)

Prop

\(\forall \omega \text{ and for all clearing schedule } \pi,\)

\[
\liminf_T \frac{V_T^{(LP)}(\omega)}{T} \leq \liminf_T \frac{V_T^\pi(\omega)}{T} \tag{6}
\]
Proof Sketch II: Discretise Arrivals using Rate Ratios $\rho_\ell$

Given $\varepsilon > 0$, choose intervals,

$\mathcal{N}_{\ell}^{(\ell,n)}$ arrivals in interval $n$ for pico $\ell$ mean $m_\ell(n)$

For all $0 < \delta < 1/2$ there exists $l_{n,\ell} > 0$

$$\mathbb{P}\left\{ \frac{1}{T} \mathcal{N}_{\ell}^{(\ell,n)} \notin [(1 - \delta)m_\ell(n), (1 + \delta)m_\ell(n)] \right\} \leq e^{-Tl_{n,\ell}} \quad (7)$$

Borel-Cantelli implies $\exists T_E$ all arrivals close to expectation, $\forall T > T_E$
Finite set $A$ of rate ratio policies,

$$\liminf_{T} \frac{V_{T}^{(LP)}}{T} \geq \liminf_{T} \inf_{a \in A} \frac{V_{a}^{a}}{T} - \frac{L \varepsilon D}{R}$$

(8)

$$= \inf_{a \in A} \liminf_{T} \frac{V_{a}^{a}}{T} - \frac{L \varepsilon D}{R}$$

(9)

$$\geq (1 + \eta) - \frac{L \varepsilon D}{R}$$

(10)
Utility Scheduling and Stability
Modelling Assumptions

- Discrete set - location $k$ in cell $\ell$ - $(k, \ell)$, $k = 1, \cdots, K_\ell$
Modelling Assumptions

- Discrete set - location $k$ in cell $\ell$ - $(k, \ell), \ k = 1, \cdots, K_\ell$
- Unit exponential files
Modelling Assumptions

- Discrete set - location $k$ in cell $\ell - (k, \ell), \ k = 1, \cdots, K_l$
- Unit exponential files
- Independent Poisson streams, $\lambda^{(\ell)}_k > 0$
Modelling Assumptions

- Discrete set - location $k$ in cell $\ell - (k, \ell), \ k = 1, \ldots, K_l$
- Unit exponential files
- Independent Poisson streams, $\lambda^{(\ell)}_k > 0$
- Physical Rates $R^{(\ell)}_k$ pico, $S^{(\ell)}_k$ macro
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Time Sharing

Time Sharing Vector \((a, b)\)
Time Sharing

Time Sharing Vector \((a, b)\)

Feasibility constraints,

\[
\sum_{k=1}^{K^{(\ell)}} a_k^{(\ell)} + \sum_{m=0}^{L} \sum_{k=1}^{K^{(m)}} b_k^{(m)} \leq 1, \quad \forall \ell. \tag{11}
\]

with throughput,

\[
T_k^{(\ell)} = a_k^{(\ell)} R_k^{(\ell)} + b_k^{(\ell)} S_k^{(\ell)} \tag{12}
\]
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Processor Sharing Model for a HetNet

1
2
3

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Whiting
Heterogeneous Networks
Stability Region $\Lambda_0$

\[ \mathcal{T} \doteq \{(a, b) : (a, b) \text{ feasible}\}, \]

\[ \Lambda \doteq \bigcup \{ \mathcal{T} (a, b) : (a, b) \in \mathcal{T}\} \]

Then,

\[ \Lambda_0 \doteq \{ \lambda : \exists \varepsilon > 0, \lambda + \varepsilon \in \Lambda\} \]

Stable scheduler exists iff $\lambda \in \Lambda_0$
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Continuous Time Markov Processes

\[ \mathbf{N}(t) = \left( \mathbf{N}^{(0)}(t), \ldots, \mathbf{N}^{(L)}(t) \right) \in \prod_{\ell} \mathbb{N}^{K_{\ell}}_0 =: \mathcal{N} \]

Arrivals, rate \( \lambda^{(\ell)}_k \),

\[ \left( \mathbf{N}^{(0)}, \ldots, \mathbf{N}^{(L)} \right) \rightarrow \left( \mathbf{N}^{(0)}, \ldots, \mathbf{N}^{(L)} \right) + \left( 0, \ldots, e_k^{(\ell)}, \ldots, 0 \right) \]

Departures policy \( \theta \), in state \( \mathbf{n} \in \mathcal{N} \), rate \( T (a^\theta(n), b^\theta(n)) \)

\[ \left( \mathbf{N}^{(0)}, \ldots, \mathbf{N}^{(L)} \right) \rightarrow \left( \mathbf{N}^{(0)}, \ldots, \mathbf{N}^{(L)} \right) - \left( 0, \ldots, e_k^{(\ell)}, \ldots, 0 \right) \]
A Static Utility Optimization Problem

\[ U(a, b) = \sum_{\ell=0}^{L} \sum_{k} N_{k}^{(\ell)} U_{\alpha} \left( \frac{T_{k}^{(\ell)}(a, b)}{N_{k}^{(\ell)}} \right) \]  

(13)

\( \alpha \)-fair utilities

\[ U_{\alpha}(\cdot) = (1 - \alpha)^{-1}x^{1-\alpha}, \ \alpha \in (0, \infty) \]

For solution to above,, see [2]

Prop (Hanly, W.)

Suppose $\lambda \in \Lambda_0$. Then $\forall \alpha > 0$ the Markov Process defined by $\alpha$-fair scheduling is positive recurrent so that

$$\mathbb{P}\{N(t) = N\} \to \pi^\alpha(N) \text{ as } t \to \infty$$

(14)

Moreover limiting $\alpha$ moments exist; that is, for all $(k, \ell)$,

$$\mathbb{E}_{\pi^\alpha} \left[ \left(N_k^{(\ell)}\right)^\alpha \right] < \infty$$

(15)
Proof Sketch

As demonstrated in [3]

\[
L(N) = \sum_{\ell=0}^{L} \sum_{k=1}^{K(\ell)} \left\{ \lambda_k^{(\ell)} \right\} - \alpha \left\{ N_k^{(\ell)} \right\}^{1+\alpha} \left(1 + \alpha\right)
\]

is a Lyapunov function

Let \( N(n) \) jump chain sequence of the uniformized Markov process, then,

\[
L(N(n))
\]

has supermart. property outside a compact set.

Example: Proportional Fair Scheduler

\[ U_N \triangleq \sum_{\ell=0}^{L} \sum_{k=1}^{K^{(\ell)}} N_k^{(\ell)} \log \frac{T_k^{(\ell)}}{N_k^{(\ell)}} \]  

(17)

Quadratic Lyapunov function \( L \),

\[ L(N) \triangleq \frac{1}{2} \sum_{\ell=0}^{L} \sum_{k=1}^{K^{(\ell)}} \frac{\left\{ N_k^{(\ell)} \right\}^2}{\lambda_k^{(\ell)}} \]  

(18)
Numerical Results

![Graph showing sample mean N_k/T over iterations (T) for different network types: Macro, Pico 1,1, Pico 2,1, Pico 1,2, and Pico 2,2. The x-axis represents iterations (T) and the y-axis represents the sample mean N_k/T.](image)
Conclusions

- Traffic Capacity Determined by LP
- Fixed Schedule Stable
  - Estimate $\eta$
  - Estimate $R_\ell(\xi), S_\ell(\xi)$
  - Infer Capacity
- Results extend to more general networks
Conclusions

- Traffic Capacity Determined by LP
- Fixed Schedule Stable
  - Estimate $\eta$
  - Estimate $R_\ell(\xi), S_\ell(\xi)$
  - Infer Capacity
- Results extend to more general networks
- $\alpha$-fair Utility Scheduler maximally stable
- Equilibrium Moments shown to exist depending on $\alpha$
- Results extend to periodic schedulers
Thanks!